

SOLVING VERY LARGE-SCALE SECURITY-CONSTRAINED OPTIMAL POWER FLOW PROBLEMS BY COMBINING ITERATIVE CONTINGENCY SELECTION AND NETWORK COMPRESSION

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Abstract – This paper proposes a practical algorithm for solving very large-scale SCOPF problems, based on the combination of a contingency filtering scheme, used to identify the binding contingencies at the optimum, and a network compression method, used to reduce the complexity of the post-contingency models included in the SCOPF formulation. By combining these two complementary simplifications, it is possible to solve SCOPF problems addressing both preventive and corrective controls on continental sized power system models and with a very large number of contingencies. The proposed algorithms are implemented with state-of-the-art solvers and applied on a model of the European transmission system, of about 15000 buses, and with about 11000 contingencies.

Keywords: *Security-constrained optimal power flow, contingency filtering, network compression, network equivalents, nonlinear programming*

1 INTRODUCTION

1.1 Motivation and related work

The security-constrained optimal power flow (SCOPF) problem is in its general form a nonlinear, non-convex, static, large-scale optimization problem with both continuous and discrete variables [1]. The efficient solution of SCOPF problems is indispensable for system operators, in the context of planning, operational planning and real-time operation. SCOPF problems have been formulated in various settings, in particular in the “preventive only” mode [1] and in the “corrective also” mode [2], the difference between these modes being that the former does not consider the possibility of re-scheduling controls in post-contingency states, except for automatic responses to contingencies (e.g., generators participation in frequency control, automatic tap-changers, etc).

One of the main challenges of SCOPF problems is their huge dimensionality, especially when they are formulated for very large-scale systems and/or when a very large number of contingencies have to be considered. The direct solution of these problems, for very large-scale power systems, would indeed imply the simultaneous representation of a very large number of network constraints multiplied by a very large number of post-contingency cases, and hence would lead to extreme memory requirements and totally prohibitive computing times.

However, in real-life applications a very large proportion of the candidate contingencies are generally not constraining the SCOPF problem, and for most of the contingencies, most of the network constraints are inactive. This sparse nature of the problem suggests that SCOPF calculations could be applied effectively to very large-scale power systems, if they could benefit from contingency selection and network reduction methods.

In the SCOPF literature, three main classes of approaches have already been proposed in order to mitigate the drawbacks of the direct approach: (i) iterative contingency selection schemes [1,3,4,5,6,7], (ii) decomposition methods [2,12], and (iii) network compression (NC) [8].

The first class of approaches generally rely on the following ingredients: (i) a SCOPF solver which is applied to small subsets of potentially active contingencies, (ii) a (steady-state) security analysis engine (SA) used to globally check N-1 security, (iii) a contingency filtering (CF) technique, and (iv) an OPF formulation (and solver) used to check whether post-contingency states may be secured via corrective control [1,3,6,7,8]. Furthermore, SCOPF computations could also be simplified by adding to the base case constraints only a small subset of relevant post-contingency inequality constraints (e.g. by using linear approximations around a base case) and by neglecting in the first place all post-contingency equality constraints [1,4,5].

The second class of approaches iterate between a certain number of slave sub-problems (post-contingency cases) and the master problem (pre-contingency case). The sub-problems receive the optimal values of pre-contingency control variables at the current iteration from the master problem; in turn they send a linear constraint to the master problem. The latter accumulates these linear constraints beside the base case constraints. The iteration proceeds until an optimality criterion is satisfied. The convergence of the decomposition algorithms can only be guaranteed under convexity assumptions which may hold only for the linearized model but can not be proven for the AC model [2]. As a consequence, for the latter, the decomposition methods may fail or produce a sequence that does not converge to a (local) optimum. The use of the decomposition strategies (Benders, Dantzig-Wolfe, Talukdar-Giras, etc.) for large scale optimization problems have been investigated by Lasdon in [12].

The third class of approaches proposes a network compression of post-contingency states which allows reducing the size and the CPU times of the post-contingency constraints. This technique exploits the property that the impact of a contingency is generally limited to a localized area of the network.

Each of these approaches required to solve a SCOPF modeled as an optimization problem. The following state-of-the-art methods are often used: sequential linear programming based methods [1,4,5], Newton methods [2,3], and interior-point methods [6,7,8].

Although several commercial SCOPF packages are available from various vendors, and are routinely used by many system operators, the scientific literature reporting on experiments using SCOPF solvers on very large-scale systems is still quite limited. Indeed most publications reporting on SCOPF algorithms provide only results obtained on small to medium sized power systems, rarely exceeding 1000 buses. Furthermore, those publications that report results on larger power system models typically rely on questionable models (e.g. the DC power flow approximation [5]) or on simplified solution techniques (e.g. successive linear programming [1,4,5]), which strongly limit their practical impact (e.g. to the optimization of active power flows only under mildly loaded conditions).

1.2 Paper contribution and organization

In this paper we propose a unified approach and algorithms for solving the SCOPF problem for very large-scale systems. This approach combines an iterative contingency filtering (CF) algorithm, as proposed in [6,7], with a network compression (NC) method, as proposed in [8]. Our algorithm thereby acts simultaneously along two complementary directions to reduce the overall complexity of the SCOPF problem. Indeed, on the one hand, the CF technique is used to quickly identify the binding contingencies at the optimum (and hence limit the number of contingencies included in the SCOPF computations carried out at successive iterations), and, on the other hand, the NC method is used in order to reduce the size of each post-contingency model included in the SCOPF computations, by identifying the potentially affected areas for each contingency and by representing only these latter as SCOPF constraints. In the rest of this paper we will use the acronym ISCOF-NC, in order to denote our approach (iterative security-constrained optimal power flow with network compression)

Importantly, and in contrast to most approaches applied to very large-scale SCOPF problems, the implementation of our ISCOF-NC uses a non-linear AC network model in both pre-contingency and post-contingency states. It exploits also recent developments in the context of nonlinear programming solvers [10]. To cope with corrective control actions, we have extended the original NC approach, which was designed for “preventive only” SCOPF problems [8].

We use a quite challenging optimization problem to demonstrate the approach, namely a “corrective also” SCOPF problem for the whole European Transmission System (ETN) (containing about 15000 buses) with respect to about 11000 contingencies.

The rest of the paper is organized as follows. Section 2 provides the general formulation of the SCOPF problem, Section 3 details the proposed algorithms, Section 4 reports numerical results, and Section 5 concludes.

2 GENERAL SCOPF FORMULATION

The general formulation of the SCOPF problem that we address in this paper can be compactly formulated in the following way [1,2]:

$$\begin{aligned} & \min f(x_0, u_0) \\ & s.t. \begin{cases} g_0(x_0, u_0) = 0 \\ h_0(x_0, u_0) \leq 0 \\ g_k(x_k, u_k) = 0, k \in C \\ h_k(x_k, u_k) \leq 0, k \in C \\ -\tau_{rest} \left[\frac{du_k}{d\tau} \right]_{\max} \leq u_k - u_0 - \Delta u_k \leq \tau_{rest} \left[\frac{du_k}{d\tau} \right]_{\max}, k \in C \end{cases} \end{aligned} \quad (1)$$

where C is the set of postulated contingencies, subscript k refers to variables and constraints of the k^{th} post-contingency state, subscript “0” refers to variables and constraints of the base case (pre-contingency state), x and u denote the vectors of state and control variables, $f(x, u)$ is the objective function, $g(x, u)$ and $h(x, u)$ denote the vectors of equality and inequality constraints, Δu_k is the vector of changes in control variables due to the automatic control of the system following the k^{th} contingency, τ_{rest} is the allowed restoration time (equal to 0 in “preventive only” approach) and $\left[\frac{du_k}{d\tau} \right]_{\max}$ is the vector of maximum possible rates of change of control variables.

Notice that in this paper we will not focus on the optimal treatment of discrete variables, and hence we will model them as continuous variables. The formulation (1) corresponds thus to a so-called Non-Linear Programming (NLP) problem.

3 ITERATIVE SCOPF WITH NETWORK COMPRESSION (ISCOF-NC)

3.1 Global scheme of the proposed approach

The flowchart represented at Figure 1 gives a global overview of the proposed algorithm.

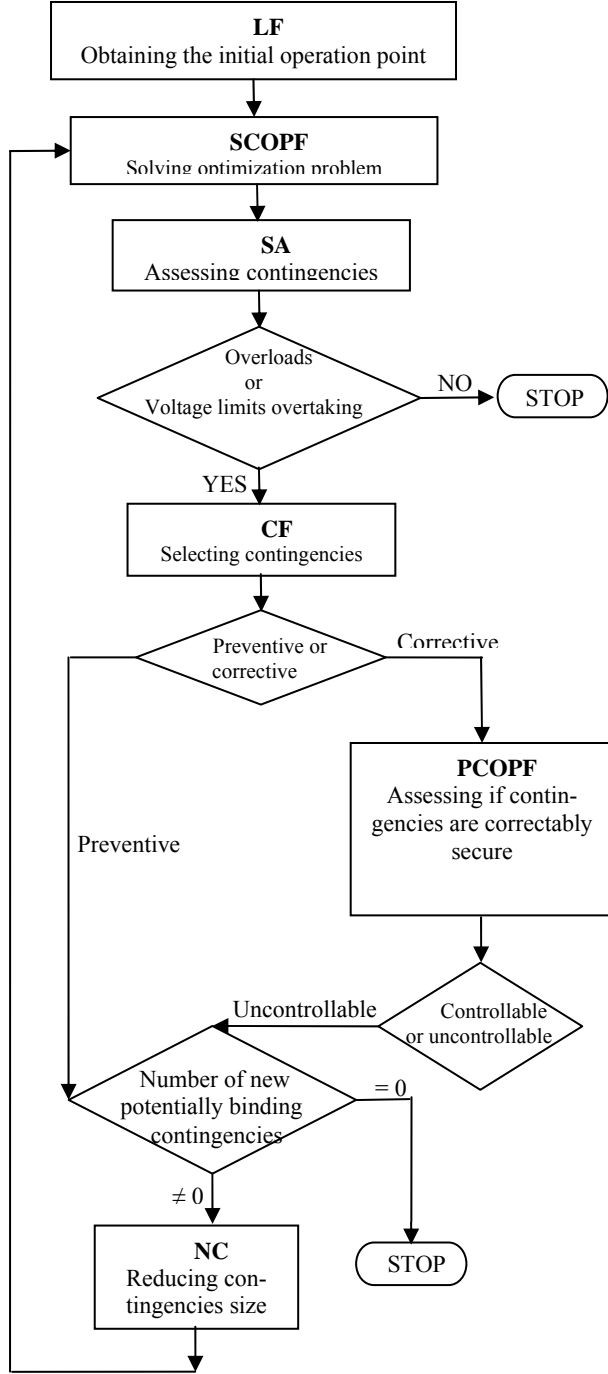


Figure 1: ISCOF-NC Flowchart

3.2 Overview of the computational modules used by the ISCOF-NC approach

The ISCOF-NC uses the following modules:

- A Load flow (**LF**) computation module is used to obtain a reasonable starting point of the ISCOF-NC algorithm.
- A **SCOPF** module is used to solve the main SCOPF problem (§2), which includes only the contingencies selected by the filter and further compressed by the NC method. Note that at the first iteration, the set of potentially binding contingencies

is empty and the SCOPF is consequently an OPF computation concerning only the pre-contingency conditions.

- A Security Analysis (**SA**) module checks whether, at the optimal solution provided by the SCOPF module, there are some among the candidate contingencies that would lead to violations of some post-contingency constraints (branch flows or voltage limits). The so identified contingencies are called *critical* contingencies. If the set of critical contingencies is empty an acceptable solution of the SCOPF problem is found and computations terminate.

Notice that because we use the network compression method within the SCOPF (see section 3.5), it may be possible that when running the SA module at the optimal solution provided by the SCOPF, some of the potentially binding contingencies (those already included in SCOPF) still lead to small constraint violations. If these violations are above a given tolerance, the compression factor used in the NC module (see section 3.5) will be reduced for the concerned contingencies for the subsequent iterations. Relevant equipments may be missed by the network compression method, for instance equipments not impacted by the incident but close to their limits. To overcome this drawback, those equipments can be included directly in the active region.

- A Contingency Filter (**CF**) module selects among the critical contingencies those that are candidates to be added to the current SCOPF problem. These contingencies are called *selected* contingencies. Further details about the CF techniques are provided in section 3.4.
- A Post-Contingency OPF (**PCOPF**) module which, in the “corrective also” approach, keeps among the selected contingencies only those unable to reach a feasible point within the restoration time. For this purpose, each selected contingency is assessed using the PCOPF module to evaluate the overall amount of restoration time needed to reach a feasible point.

For each contingency k , the PCOPF problem can be formulated as follow:

$$\begin{aligned}
 & \min \sum z_k \\
 & \begin{cases} g_k(x_k, u_k) = 0 \\ h_k(x_k, u_k) \leq 0 \end{cases} \\
 & \text{s.t.} \begin{cases} |u_k - u_0^* - \Delta u_k| \leq \tau_{rest} \left[\frac{du_k}{d\tau} \right]_{\max} + z_k \\ z_k \geq 0 \end{cases} \quad (2)
 \end{aligned}$$

Where u_0^* is the vector of base case optimal values of control variables (stemming from the optimal solution of the SCOPF problem at the current iteration)

and z_k is the vector of positive slack variables aimed to relax coupling constraints. The other variables have the same meaning as in formulation (1).

If the value of the PCOPF objective is nonzero, an extra amount of restoration time is needed. The contingency is then called *uncontrollable* and is added to the SCOPF problem. Otherwise, the contingency is called *controllable* and is not added to the SCOPF problem.

The uncontrollable contingencies that must be added to the SCOPF problem are called *potentially binding* contingencies. If the set of potentially binding contingencies is empty an acceptable solution of the SCOPF problem is found and computations terminate.

Note that the PCOPF is not needed in the “preventive only” approach where all selected contingencies are obviously considered as potentially binding.

- f) For each potentially binding contingency, a Network Compression (NC) module is used in order to reduce as much as possible the size of the post-contingency power system model that has to be added to the current SCOPF problem. Further details about the NC method are provided in section 3.5.

3.3 Description of the approach algorithm

The main steps of the **ISCOF-NC** algorithm are summarized as follows (see also Figure 1):

1. Obtain an initial base case by performing a load flow calculation (**LF** module).
2. Solve the SCOPF problem by adding new potentially binding contingencies to those already identified at the previous iterations.
3. Identify critical contingencies among the full set of postulated contingencies, by using the **SA** module at the optimal solution of the current SCOPF problem. If the set of critical contingencies is empty computations terminate.
4. Select among the critical contingencies a subset of potentially binding ones by using the **CF** module.
5. In the “corrective also” approach, identify the uncontrollable contingencies among the selected ones, by means of the PCOPF module.
6. Define the set of additional potentially binding contingencies: in “preventive-only” case, those selected by the CF; in “corrective also” case, the subset of contingencies selected by the CF identified as uncontrollable by the PCOPF.
7. Reduce the model of each new potentially binding contingency by using the **NC** module. Note that unsatisfactory compressed models of potentially binding contingencies added in previous iterations will be reviewed, eventually with tightened tolerances (cf. §3.2).

8. As long as the subset of new potentially binding contingencies is not empty, these contingencies are added in the compressed form to the current SCOPF problem and the iterative process starts again from step 2.

3.4 Contingency filtering techniques

The Contingency Filter selects a subset of the critical contingencies using one of the following two approaches: the non-dominated contingency (NDC) approach [6,7], or the security index (SI) approach [1,3,4,6,7]. Both approaches exploit the amount of constraints violation at the converged load flow solution obtained from the SA module after assessing all given contingencies at a given iteration.

The proposed algorithm focuses on the NDC approach. A contingency k is *dominated* by contingency j if contingency j leads to a larger or equal violation for every constraint than contingency k , and a strictly larger violation for at least one constraint. Hence, a contingency is *non-dominated* if no other contingency dominates it. The principle of the NDC contingency filtering approach is to select the non-dominated contingencies among the given set of contingencies.

3.5 Network compression method

3.5.1 Contingency compression concept

The compression method [8] identifies, for each postulated contingency, a limited area called the active region where the contingency has a significant impact. The variables and elements of the active region are kept in their real identity. The nodes and elements that do not belong to the active region are replaced by a REI-DIMO equivalent network [9].

The active region is composed of two sub regions: the direct and the indirect regions (Figure 2).

The direct active region is defined as the set of buses and branches where the contingency has a significant impact in terms of voltages and power flows deviations with respect to base case values.

The indirect active region concept is motivated by the fact that there are control variables located outside the direct active region that may significantly impact (during the optimization process) the constraints concerning the elements selected in the direct active region.

The criteria and relations concerning the setting up of the direct and indirect active region are described in details in [8]. However, the possible impact of the corrective actions should be taken into account while determining the active region. The extension of the NC approach to the corrective SCOPF is accomplished in the following way:

- The indirect active region will be enlarged with all the nodes and branches having their voltages and power flows significantly impacted by the *variations of the controls selected initially in this region*. This choice aims to avoid that varying indirect area

controls to remove violated constraints would in turn violate constraints *outside* the direct active area.

- When generators active powers are used as controls, it is mandatory to include in addition in the indirect active region the connection nodes of these generators as well as the branches having the flows significantly impacted by the variations of these active powers in the corresponding post-contingency states. These branches are selected in the following manner:
- The impact on power flows at both sides of each branch is estimated by means of the sensitivity matrix relating the variations of the active and reactive branch power flows ($\Delta T_P, \Delta T_Q$) to the active and reactive power injections ($\Delta P, \Delta Q$) [8]:

$$\begin{bmatrix} \Delta T_P \\ \Delta T_Q \end{bmatrix} = - \begin{bmatrix} \frac{\partial T_P}{\partial \theta} & \frac{\partial T_P}{\partial V} \\ \frac{\partial T_Q}{\partial \theta} & \frac{\partial T_Q}{\partial V} \end{bmatrix}^{(k)} [J^{(k)}]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}, \quad (3)$$

or equivalently:

$$\begin{bmatrix} \Delta T_P \\ \Delta T_Q \end{bmatrix} = [STsP]^{(k)} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (4)$$

where $[J^{(k)}]$ is the Jacobian matrix.

The superscript (k) indicate that the elements of the matrices are evaluated for the post-contingency state k (both topology and voltage values). They should therefore be updated with the estimation of the post-contingency voltage values.

The terms of the sensitivity matrix $[STsP]^{(k)}$

are: $\frac{\partial T_P}{\partial P}, \frac{\partial T_Q}{\partial P}, \frac{\partial T_P}{\partial Q}, \frac{\partial T_Q}{\partial Q}$. Only the ap-

parent power flow sensitivities at both sides of each branch with respect to the active power injections will be used, according to the relation:

$$\frac{\partial T}{\partial P} = \text{sign} \left(\frac{\partial T_P}{\partial P} \right) \sqrt{\left(\frac{\partial T_P}{\partial P} \right)^2 + \left(\frac{\partial T_Q}{\partial P} \right)^2} \quad (5)$$

- The branch ik located outside the direct active region is added to the indirect active region if its scaled apparent power flow sensitivities with regard to all pair combinations ($P_{g,j}, P_{g,l}$) of active power generations selected as controls, injected in the nodes j and l met the following requirement:

$$\max_{\text{all}(j,l)} \left(\left| \left(\frac{\partial T_{ik}}{\partial P_{g,j}} \right)_N - \left(\frac{\partial T_{ik}}{\partial P_{g,l}} \right)_N \right| \right) \geq \varepsilon_T \quad (6)$$

where ε_T is the threshold value and the subscript N indicates that the power flow sensitivities are scaled (normalized).

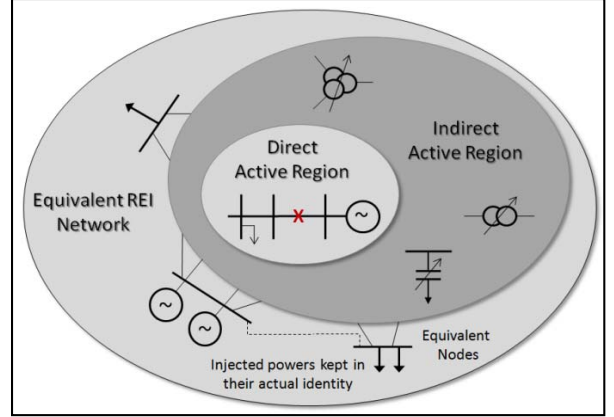


Figure 2: Network compression method concept

3.5.2 Network reduction

The direct and indirect active regions compose the active region of a contingency and must therefore be kept in their real identity. The nodes that do not belong to the active region are reduced using the REI-DIMO equivalencing method [8,9].

The generating units (respectively loads) that do not belong to the active region are grouped in one or several *REI-DIMO generator* (respectively *REI-DIMO load*) equivalent nodes. The generators (respectively loads) are grouped according to their coherency evaluated on the basis of the phase of their injected current.

The generators and loads connected at the *REI-generator* and *REI-load* equivalent nodes keep their real identity. Consequently the control variables attached to these generators and loads (namely their active and reactive powers) continue to exist as such in the optimization problem although they belong to the equivalent network.

4 NUMERICAL RESULTS

4.1 Description of the test system

In this section we apply the proposed approach on a very large system stemming from the whole European transmission system, model built in the context of PEGASE project [13,14].

This system contains around 15000 buses and 4500 generators. We postulate a set of 11265 contingencies which consists in the N-1 loss of any line, transformer, generator or capacitor bank connected to the 380kV voltage level of the network. The initial solution satis-

fies the production limits (both active and reactive powers).

4.2 Description of the SCOPF problem

With respect to the SCOPF formulation (1), the objective of the problem is to minimize the shift of active power production needed to preventively ensure the N-1 security of the system. The control variables are the generators active and reactive powers. The constraints are the AC power flow equations, branches flow limits and physical bounds on generators active and reactive powers.

As each considered state brings approximately 45000 variables, the resolution of a SCOPF containing the whole 11265 contingencies in full AC representation is obviously prohibitive in terms of memory resources and computation time (≈ 500 million variables).

Our simulations highlight the filtering and compression ratios obtained respectively by the CF and NC modules and the expected size of the reduced SCOPF. Some comparisons of computation time obtained on optimizations solved by the SCOPF module are also given.

All tests have been performed on a PC 2.4-GHz Intel Core Duo P8600 with 1.9-Gb RAM.

4.3 Security Assessment and Contingency Filter

For the purpose of this example, the Security Assessment monitors only the current on each branch of the system and the Contingency Filter uses the non-dominancy criterion.

Since we started from a situation where some transmission system elements are already overloaded in the base case, all the 11265 postulated contingencies have naturally been identified as critical by the Security Assessment.

Nevertheless, the Contingency Filter finally selects only 275 non-dominated contingencies, hence only 2.5% of the set of postulated contingencies.

Despite the very good global problem complexity reduction at this stage (i.e. 97.5% with respect to the direct SCOPF approach), the problem is still too large for the available computational resources (≈ 15 million variables).

4.4 Network Compression

For the purpose of this example, the Network Compression uses the most restrictive tolerances found in [8], namely:

- 1.5 degree for $\varepsilon_{\theta,dir}$ and $\varepsilon_{\theta,ind}$
- 1.5% for $\varepsilon_{V,dir}$ and $\varepsilon_{V,ind}$
- 3% for $\varepsilon_{T,dir}$
- 5% for $\varepsilon_{T,ind}$

Where $\varepsilon_{\theta,dir}$, $\varepsilon_{V,dir}$, $\varepsilon_{T,dir}$ (respectively $\varepsilon_{\theta,ind}$, $\varepsilon_{V,ind}$, $\varepsilon_{T,ind}$) are tolerances on voltage angle, voltage magnitude and apparent flow on branches for determining the

Direct Active Region (respectively the Indirect Active Region).

The *compression factor* for a single contingency is defined as the number of eliminated busses by the NC module divided by the total number of busses in the original system divided by, expressed in percent.

On the set of 275 selected contingencies, the NC module obtains a rough compression factor of 98% leading to an average number of 300 busses for characterizing the model of each kept contingency.

For most selected contingencies (i.e. 247 cases), less than 2% of the original network is kept, which represents an excellent compression ratio given the tight tolerances used. On the other hand, we have also identified 7 contingencies for which more than 30% of the original network is kept, and up to almost the whole network for three of them.

A deeper analysis of these contingencies shows that they effectively provoke strong perturbations of the system.

The remaining 21 contingencies lead to a compression factor between 90% and 98%.

The global compression factor of 98% leads to a SCOPF problem with around 300000 variables which can in principle be handled by the SCOPF module. However, the tight memory limitations imposed in our simulations (less than 2GB of RAM) prevents us to include all the 275 contingencies in compact representation and more than 2 contingencies in full representation. Consequently, the SCOPF calculation is in the next stage limited to the first 260 contingencies in the decreasing order of compression factor.

4.5 SCOPF solution

Three similar SCOPF problems are built from the initial test case.

Problem A contains the set of compressed contingencies after the application of CF and NC. As mention in the previous section only the first 260 contingencies in terms of compression factor have been considered.

Problem B includes the compressed model of two among the 260 contingencies of problem A.

Problem C includes the full AC model of the same two contingencies as problem B.

The three SCOPF problems are compared in terms of number of variables, number of constraints and computation time.

The SCOPF problems are solved using IPSO, an Integrated Power Systems Optimiser developed by Tractebel Engineering [11].

Table 1 shows the characteristics of each SCOPF problem:

SCOPF	#contingencies	#variables	#constraints
A	260	94872	81628
B	2	41796	30778
C	2	115370	91344

Table 1: Problems characteristics

Table 2 shows the computation time to solve the core optimisation of each SCOPF problem:

SCOPF	Elapsed Computation time (s)
A	663
B	219
C	853

Table 2: Computation time

The accuracy of both methods used in the present algorithm have been discussed in previous contributions [6,7,8].

5 CONCLUSIONS

The feasibility of the approach has been proven under very stringent computational limitations (memory and CPU resources). Dedicated architectures certainly allow to handle larger SCOPF problems and to improve computation time.

However, in the context of day-ahead operational planning, the proposed algorithm is already useful and the results very encouraging for comfortable computations of SCOPF on very large scale systems.

For more stringent conditions (N-x or real-time), the proposed algorithm may be adapted to provide a mean to reduce risk. For instance, by selecting the largest number of the most critical contingencies (e.g. sorted by decreasing order of compression factor) such that the problem is still compatible with real-time requirements, the ISCOF-NC can provide indications to improve the security of the system. Moreover, as in real-time the operating state is not likely to differ drastically from one time-window to the other, the ISCOF-NC could use the information of the previous solution (e.g. the solution itself, the list of binding contingencies) to start quicker than “from scratch” assumptions.

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